Synthesis of Even-Order Equally Terminated Transmission-Line Bandstop Filters

RAMAMURTY NEDUNURI, MEMBER, IEEE

Abstract—**An exact synthesis procedure for the design of even-order equally terminated transmission-line bandstop filters is described. The method is optimum in the sense that it does not introduce redundant unit elements into the network. The final network is obtained in the form of a cascade of unit elements and shunt open-circuited stubs.**

I. Introduction

An EXACT method for the design of microwave bandstop filters has been described by Schiffman and Matthaei [1]. The method starts with the transmission-line equivalent of the lumped-element low-pass filter network, and then uses Kuroda's transformation to introduce additional unit elements into the network without affecting the attenuation response of the filter.

In [2] Gupta and Wenzel have given another procedure for the exact design of odd-order bandstop filters. Starting with the transfer function of the filter network, the final cascaded unit element-stub network is obtained by an exact synthesis procedure. The method is optimum in the sense that it does not introduce redundant unit elements into the network.

In this paper we describe an exact synthesis procedure for the design of even-order equally terminated bandstop filters. The method is also optimum because no redundant unit elements are introduced into the network.

II. Synthesis Procedure

The first step in the synthesis procedure is to find a transfer function which gives a Chebyshev-type attenuation response, and which is realizable in the form of a cascade of q unit elements and q shunt open-circuited stubs. In [3] Carlin and Kohler showed that the transfer function:

$$|s_{12}|^2=\frac{1}{1+\epsilon^2F_n^2(x)}$$

where

$$F_n(x)=\cos\left[q\phi+(n-q)\theta\right]$$
$$\phi=\cos^{-1}x$$
$$\theta=\cos^{-1}\left[x\sqrt{\frac{\alpha^2-1}{\alpha^2-x^2}}\right]$$

n is the order of the filter and the total number of network elements; and

$$x=\alpha\sin\left(\frac{\pi}{2}\cdot\frac{\omega}{\omega_0}\right),\quad -\alpha\leqslant x\leqslant\alpha$$

gives a Chebyshev-type attenuation response, and that it is realizable in the form of a cascade of q unit elements and $n-q$ shunt open-circuited stubs. Here we are interested in the special case $n=2q$. However, since n is even, the filter network will have unequal terminating resistances. To obtain equal-resistance terminations, it is necessary to use a frequency transformation [4], [5]. We shall now derive such a transformation. Let n be even and let $x_{\min}$ be the smallest zero of the function $F_n(x)$. Then the transformation is given by

$$y^2=\frac{x^2-x_{\min}^2}{1-x_{\min}^2}.$$

We have $F_n(x)=0$ if

$$\cos q(\phi+\theta)=0$$

or

$$\phi+\theta=(2i-1)\pi/n,\quad \text{for } i=1,2,\cdots,n/2.$$

Substituting for ϕ and θ, we obtain

$$\cos^{-1}x+\cos^{-1}\left[x\sqrt{\frac{\alpha^2-1}{\alpha^2-x^2}}\right]=\frac{(2i-1)\pi}{n}$$

or

$$\frac{(\alpha+\sqrt{\alpha^2-1})x^2-\alpha}{\sqrt{\alpha^2-x^2}}=\cos\frac{(2i-1)\pi}{n}.$$

Let

$$m_i=\cos\frac{(2i-1)\pi}{n},\quad \text{for } i=1,2,\cdots,n/2.$$

Then we have

$$x_i^2=\frac{b-m_i^2+m_i\sqrt{c+m_i^2}}{2d}$$

where

$$b=2\alpha(\alpha+\sqrt{\alpha^2-1})$$
$$c=4\alpha\sqrt{\alpha^2-1}\,(\alpha+\sqrt{\alpha^2-1})^2$$
$$d=(\alpha+\sqrt{\alpha^2-1})^2.$$

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The author is at 6, Dunkirk Rise, Rochdale, Lancashire 0L12 6UH, England.

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Now

$$x_{\min}^2 = x_{n/2}^2$$

$$= \frac{b - \cos^2(\pi/n) - \cos(\pi/n)\sqrt{c+\cos^2(\pi/n)}}{2d}. \quad (1)$$

The effect of the transformation is illustrated in Fig. 1 for the case $n=4$. Here

$$\beta^2 = \frac{\alpha^2 - x_{\min}^2}{1 - x_{\min}^2} \quad (2)$$

and

$$y = \beta \sin\left(\frac{\pi}{2} \cdot \frac{\omega}{\omega_0}\right), \quad -\beta \leqslant y \leqslant \beta.$$

Notice, however, that $x_{\min}^2$ is not explicitly determined from (1) because the value of α is not known beforehand. For synthesis, we are given the cutoff frequencies ω_1 and ω_2. Then we compute

$$\omega_0 = \frac{\omega_1 + \omega_2}{2}$$

and

$$\beta = \frac{1}{\cos\left(\frac{\pi}{2} \cdot \frac{\omega_0 - \omega_1}{\omega_0}\right)}.$$

Also, if $r_{\max}$ is the maximum permissible standing-wave ratio in the passband, we have

$$\epsilon = \frac{r_{\max} - 1}{2\sqrt{r_{\max}}}.$$

Now the problem is to calculate the value of α. From (2) we have

$$x_{\min}^2 = \frac{\beta^2 - \alpha^2}{\beta^2 - 1}. \quad (3)$$

Substituting for $x_{\min}^2$ from (1), we obtain

$$\cos(\pi/n)\left[\sqrt{c+\cos^2(\pi/n)} + \cos(\pi/n)\right]$$

$$= \frac{2\sqrt{\alpha^2-1}\,(\alpha+\sqrt{\alpha^2-1}\,)\left[\alpha(\alpha+\sqrt{\alpha^2-1}\,)-\beta^2\right]}{\beta^2-1}. \quad (4)$$

We can write (4) in the form:

$$\frac{\cos(\pi/n)}{\sqrt{c+\cos^2(\pi/n)} - \cos(\pi/n)} = \frac{\gamma-\beta^2}{2\gamma(\beta^2-1)}$$

where

$$\gamma = \alpha(\alpha+\sqrt{\alpha^2-1}\,).$$

Let

$$\delta = \frac{2\gamma(\beta^2-1)}{\gamma-\beta^2}.$$

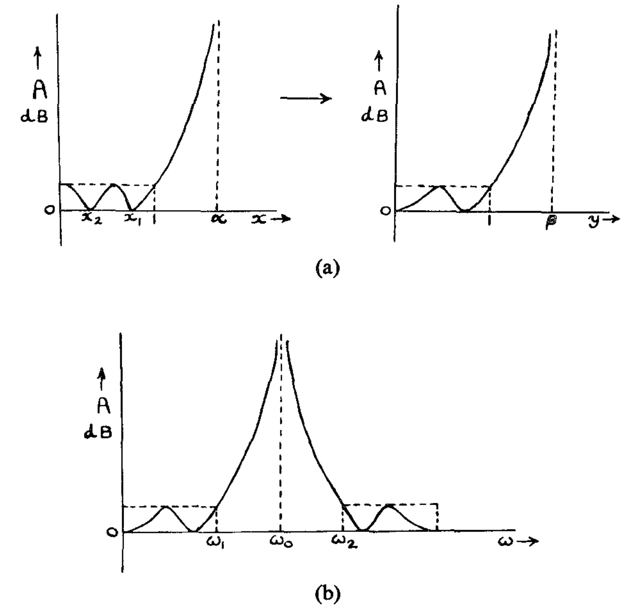

Fig. 1. (a) Low-pass type response in the normalized variable before and after frequency transformation. (b) Bandstop filter response obtained from the low-pass response of (a) by using the formula $y=\beta \sin(\frac{\pi}{2}\cdot\frac{\omega}{\omega_0})$.

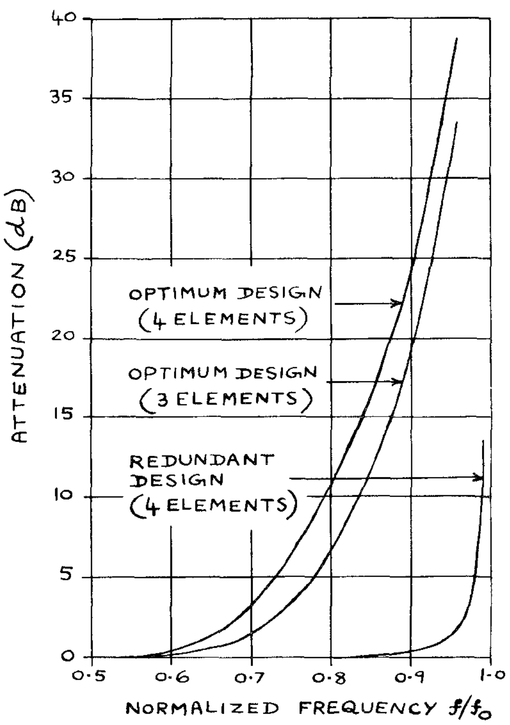

Fig. 2. Comparison of the stopband attenuation responses of various bandstop filter designs. (Maximum VSWR in the passband is 1.1 in all cases. Bandwidth is 100 percent, that is, $(f_2-f_1)/f_0=1$, where f_1 and f_2 are the cutoff frequencies.)

Then, we have

$$c + \cos^2(\pi/n) = (\delta+1)^2 \cos^2(\pi/n).$$

This gives

$$\delta(\delta+2)\cos^2(\pi/n) = c$$

or

$$\gamma-\beta^2=\beta\sqrt{\beta^2-1}\ \cos(\pi/n).$$

Substituting for γ, we obtain

$$\alpha=\frac{\mu}{\sqrt{2\mu-1}}$$

where

$$\mu=\beta\left[\beta+\sqrt{\beta^2-1}\ \cos(\pi/n)\right].$$

Once the value of α is known, we can compute $x_{\min}^2$ from (3). The modified transfer function is then given by

$$|s_{12}|^2=\left[\frac{1}{1+\epsilon^2F_n^2(x)}\right]_{x^2=(1-x_{\min}^2)y^2+x_{\min}^2}.$$

We now make the substitution

$$y^2=\frac{\beta^2\Omega^2}{1+\Omega^2}$$

where

$$\Omega=\tan\left(\frac{\pi}{2}\cdot\frac{\omega}{\omega_0}\right)$$

and synthesize the network in a straightforward manner [6]. (As a matter of fact, the synthesis is somewhat easier in the even-order case because we can derive explicit formulas for the poles and zeros of the reflection factor $s_{11}(\lambda)$.)

III. Some Additional Comments

In [2] Gupta and Wenzel have shown that in the wide-band odd-order case, the optimum design method yields a network with a substantially improved performance in the stopband when compared with the redundant design method of [1]. The same conclusion holds in the case of wide-band even-order filters also. As an example, consider the case of a 4-element 100-percent bandwidth filter designed by the present method (maximum VSWR in the passband is 1.1). The stopband attenuation response of the filter is shown in Fig. 2. For comparison the attenuation responses of a 3-element optimum filter and a 4-element redundant unit-element filter are also shown. Clearly, the 4-element optimum filter gives an improved performance in the stopband.

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Interdigital Microstrip Circuit Parameters Using Empirical Formulas and Simplified Model

ARNE BREJNING DALBY

***Abstract*—Empirical formulas are given for single- and multiple-coupled microstrips, directly giving propagation mode admittances and phase velocities to be used in simplified expressions for the admittance parameters for the 2*N*-port network in the form of *N*-coupled strips, thus forming the basis for both analysis and synthesis of interdigital microstrip circuits. Calculated and measured results are presented for two interdigital bandpass filters synthesized as Chebyshev filters.**

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The author is with the Engineering Academy of Denmark, Department of Electrical Engineering, 2800 Lyngby, Denmark.

I. Introduction

IN THE ANALYSIS and synthesis of coupled microstrip circuits, a major problem is how to obtain the primary parameters, that is, propagation mode admittances and phase velocities. For a single strip and for two identical coupled strips, formulas exist which give the propagation mode parameters as functions of the relative dielectric constant ϵ_r and the physical dimensions [2]–[4]. For multiple-coupled strips various numerical methods,

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